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An equation for the flow of a viscous liquid is obtained on the assumption that the channel has the shape of two contiguous truncated circular cones. This expression is used for determining the minimum size of micropores of nuclear filters.

In the production of nuclear microfilters [1, 2] with very small pore radius (hundreds or tens of angstroms), difficulties arise in connection with the reliable determination of their dimensions. Pore dimensions are usually determined by Poiseuille's formula derived on the assumption that the channel has cylindrical shape. However, with small radii, the shape of the channels differs considerably from the cylindrical.

The shape of the micropores of nuclear filters is determined by the ratio of the etching rate along the track $\left(V_{t}\right)$, the rate of etching through the inner surface of the track $\left(V_{G}\right)$, and the rate of etching through the film surface $\left(V_{G_{1}}\right)$ [3]. The values of $V_{t}, V_{G}$, and $V_{G_{1}}$ depend on the material of the film, the conditions of ion and ultraviolet irradiation, and on the etching technology. Under the conditions of the technology used by us [1], the mean values over the track for $V_{t}$ was $0.5 \mu \mathrm{~m} / \mathrm{min}$, and $V_{G} \approx V_{G_{1}}=0.005 \mu \mathrm{~m} / \mathrm{min}$. The change in rate due to a change in braking losses along the track does not exceed $30-35 \%$. The approximate constancy of $V_{t}$ along the track is also confirmed by the fact that the time of etching through holes in the films is proportional to their thickness.

Taking $V_{t}$ and $V_{G}$ as constant along the track and assuming that etching proceeds from both sides of the film, we have (Fig. la)

$$
\begin{equation*}
r_{2} \simeq r_{1}+\frac{l}{2} \frac{V_{Q}}{V_{t}} \simeq r_{1}+\frac{l}{2} \operatorname{tg} \vartheta_{0} \tag{1}
\end{equation*}
$$

In our experiments $Z=10 \mu \mathrm{~m}, V_{G} / V_{t} \simeq 0.01$, and thus $r_{2}$ differs from $r_{1}$ by $500 \AA$. For holes with $r \approx 10^{4} \mathrm{~A}$, this difference is not great, but with $r_{1}=100 \AA$, $r_{2}$ is equal to 600 $\AA$, and the assumption that the hole is cylindrical becomes untenable. In that case, using Poiseuille's formula leads to the determination of some effective radius ( $r_{e f}$ ) differing from the minimum radius ( $r_{1}$ ) which also determines the size of the particles passing through the filter. Since the value of $\left(r_{e f}-r_{1}\right) / r_{1}$ increases with decreasing $r_{i}$, the use of the ordinary Poiseuille formula in the case of micropores with small radius leads to large errors in determining $r$.

In the case of a conical shape of the micropores, the value of $r_{1}$ can be determined by solving the problem of flow of a viscous liquid through a right circular cone (Fig. 1b). For steady-state slow flow of a liquid through a cone, Happel and Brenner [4] obtained an expression for detemining the pressure at an arbitrary point with the polar coordinates $\rho$ and $\vartheta$ :

$$
\begin{equation*}
P=P_{\infty}-\frac{\mu q}{\pi \rho^{3}} \frac{1-3 \xi^{2}}{\left(1+2 \xi_{0}\right)\left(1-\xi_{0}\right)} \tag{2}
\end{equation*}
$$

where $P_{\infty}$ is the pressure at infinity; $\xi=\cos \hat{\vartheta}, \xi_{0}=\cos \mathcal{V}_{0}$. Neglecting the edge effects, we find, on the basis of expression (2), an equation determining the motion of the liquid in a conical pipe of finite length. We determine the pressure for two cross sections with the coordinates $z_{i}(i=1,2)$, averaged over the area of these cross sections:

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Fig. 1


Fig. 2

Fig. 1. Shape of the micropore (a) and adopted notation (b).
Fig. 2. Example of the graphic solution of Eq. (9) for determining $r_{1}, \AA ; k=$ $1.3 \cdot 10^{-2} \mathrm{~A}^{-1}$.

$$
\begin{equation*}
\bar{P}_{i}=P_{\infty}-\frac{1}{S_{i}} \int_{S_{i}} \frac{\mu q}{\pi \rho^{3}} \frac{\left(1-3 \xi^{2}\right)}{\left(1+2 \xi_{0}\right)\left(1-\xi_{0}\right)^{2}} d S_{i} . \tag{3}
\end{equation*}
$$

Taking it that $\rho=z / \xi, S_{i}=\pi z_{i}^{2} \tan ^{2} \theta_{0}$, and $z_{i}=r_{i} / \tan \theta_{0}$, we obtain after some simple transformations that

$$
\begin{equation*}
\bar{P}_{i}=P_{\infty}+\frac{2 \mu q \sin ^{3} \theta_{0}}{\pi r_{i}^{3}\left(1+2 \xi_{0}\right)\left(1-\xi_{0}\right)^{2}} . \tag{4}
\end{equation*}
$$

For the difference in pressures of a liquid flowing through a truncated cone, we find

$$
\begin{equation*}
\Delta P_{1,2}=\bar{P}_{1}-\bar{P}_{2}=\frac{2 \mu q \sin ^{3} \theta_{0}}{\pi\left(1+2 \cos \theta_{0}\right)\left(1-\cos \theta_{0}\right)^{2}}\left[\frac{1}{r_{1}^{3}}-\frac{1}{r_{2}^{3}}\right] . \tag{5}
\end{equation*}
$$

For a double cone (Fig. la), $\mathrm{P}=2 \Delta \mathrm{P}_{1,2}$. By substituting $\Delta \mathrm{P}_{1_{1}, 2}$ from (5), we can obtain an expression for determining the minimum radius for the case of a hole having the shape illustrated in Fig. la:

$$
\begin{equation*}
P=\frac{8 \mu q l \cos ^{4} \frac{\theta_{0}}{2}}{\pi r_{1}^{4}\left(1+2 \cos \theta_{0}\right) \cos \vartheta_{0}}\left[\frac{r_{1}}{r_{2}}+\left(\frac{r_{1}}{r_{2}}\right)^{2}+\left(\frac{r_{1}}{r_{2}}\right)^{3}\right] . \tag{6}
\end{equation*}
$$

Taking it that $\tan \theta_{0} \approx \mathrm{~V}_{\mathrm{G}} / \mathrm{V}_{\mathrm{t}} \simeq 0.01$ and, consequently, $\cos \theta_{0} \simeq 1$,

$$
\left[\frac{r_{1}}{r_{2}}+\left(\frac{r_{1}}{r_{2}}\right)^{2} \div\left(\frac{r_{1}}{r_{2}}\right)^{3}\right]=f\left(\frac{r_{1}}{r_{2}}\right),
$$

we obtain from (6) that

$$
\begin{equation*}
P=\frac{8 \mu q l}{3 \pi r_{1}^{4}} f\left(\frac{r_{1}}{r_{2}}\right) . \tag{7}
\end{equation*}
$$

If Poiseuille's formula is equated with $P=8 \mu q Z / \pi r_{\text {ef }}^{4}$, it follows that

$$
\begin{equation*}
r_{\mathrm{ef}}^{4}=3 r_{1}^{4} / f\left(\frac{r_{1}}{r_{2}}\right) \tag{8}
\end{equation*}
$$

For a filter with the full number of pores $N$, the equation for determining $r_{1}$ has the form

$$
\begin{equation*}
r_{1}\left(\frac{P N}{0.849 Q \mu l}\right)^{1 / 4}=\sqrt[4]{f\left(\frac{r_{1}}{r_{2}}\right)} \tag{9}
\end{equation*}
$$

where $Q$ is the volume of the liquid flowing thorugh the filter in unit time. It is expedient to solve Eq. (9) graphically. Figure 2 shows a graph of the function $\sqrt[4]{f\left(r_{1} / r_{2}\right)}$ vs $r_{1}$ on condition $(Z / 2)\left(\mathrm{V}_{\mathrm{G}} / \mathrm{V}_{\mathrm{t}}\right)=500 \AA$ and the straight line $\mathrm{Kr}_{2}$, where $\mathrm{K}=\left(\mathrm{PN} / 0.849 \mathrm{Q}_{\mu} Z\right)^{1 / 4}$ was calculated for the case $P=0.65 \cdot 10^{5}$ poises, $N=3.8 \cdot 10^{9}, Q=10^{-10} \mathrm{~m}^{3} / \mathrm{sec}, \mu=10^{-3} \mathrm{~N} \cdot \mathrm{sec} / \mathrm{m}^{2}$, $Z=10^{-5} \mathrm{~m}$. Under these conditions, $K=1.3 \cdot 10^{8} \mathrm{~m}^{-1}=1.3 \cdot 10^{-2} \AA^{-1}$, and the value $\mathrm{r}_{1}=40^{\prime} \AA$ satisfies Eq. (9). In that case, the effective radius, determined on the assumption that the hole is of cylindrical shape, is $100 \AA$, i.e., it differs from $r_{1}$ by a factor of 2.5 .


Fig. 3


Fig. 4

Fig. 3. Graphs of the selectivity of acetate-cellulose and nuclear microfilters R , \%; D, A.

Fig. 4. Photograph of the surface of a nuclear microfilter; 15,000×.

Figure 3 shows the results of determining the selectivity ( R ) of filtering albumen, an albuminous molecule with diameter $40 \AA, 150 \AA$ long. $R=\left(C_{o}-C\right) / C_{o} \cdot 100 \%$. Curves 1 and 3 illustrate the results of measuring the selectivity of nuclear filters with different pore dimensions, made at the Ioffe Physicotechnical Institute, Academy of Sciences of the USSR; curve 2 illustrates the results of measuring selectivity with the aid of acetate-cellulose filters of the type "Vladipor." The effective dimensions of the nuclear filters used in plotting curve 1 were determined by the ordinary Poiseuille formula. In the case of curve 3 the pore dimensions were determined with the aid of Eq. (9), obtained by us, which is suitable for holes of conical shape.

As was to be expected, the selectivity of nuclear filters is much better than the selectivity of acetate-cellulose filters, but the graph of the selectivity (1) of nuclear filters is strongly shifted to the right. This last circumstance has to do with the above-mentioned fact that in the case of filters with small holes, the ordinary Poiseuille formula yields exaggerated dimensions of the holes.

It can be seen from Fig. 3 that when the micropore sizes are correctly estimated (curve 3), passage of $50 \%$ takes place with the same size holes of the nuclear filters and filters of the "Vladipor" type. Measurements of the pore radius based on the use of the formula suggested by us and derived for conical holes confirm the empirical rule of the biologists that to ensure almost complete passage of particles through a filter, the diameter of its holes must be twice the maximum particle size. To verify our assumption concerning the shape of micropores, we measured the maximum transverse dimension ( $2 r_{2}$ ) of the micropores of a nuclear filter for which $r_{1}$, determined by $E q$. (9), was found to be equal to $40 \AA$.

With an electron microscope and the use of the method of replicas [5], we obtained a photograph of the surface of this membrane with a magnification of 215,000 (Fig. 4). It was established by special experiments that the method of replicas yields the pore size at a depth of $20.3 \mu \mathrm{~m}$ below the surface. Taking this effect into account and bearing in mind the slight etching of the film ( $\sim 0.2 \mu \mathrm{~m}$ on each side), it ought to be expected that the size of a micropore on the surface (initial thickness of the film is $10.2 \mu \mathrm{~m}$ ) is $2 \mathrm{r}_{2}=80+920=$ 1000 A.

The real size of the holes on the film surface was determined by measuring and averaging 100 valises of the micropore diameters illustrated in Fig. 4. The size was found to be $1230 \pm 77 \mathrm{~A}$.

Our initial assumption that $V_{t}$ is constant does not permit us to expect coincidence between the theoretical and the experimental values of $r_{2}$ better than within the limits of $20 \%$.

Taking the real distribution of $V_{t}$ along the track into account improves the agreement between the theoretical and the experimental values of $r_{2}$.

The confornity of the graphs of selectivity and the value of $\mathrm{r}_{2}$ exp close to the calculated value unambiguously indicate that the suggested method of determining the size of micropores of nuclear filters is correct.

## NOTATION

$V_{t}$, etching rate along the track; $V_{G}$, rate of etching through the inner pore surface; $\mathrm{V}_{\mathrm{G}_{1}}$, etching rate through the film surface; 2 , film thickness (length of track); $\mathrm{r}_{1}, \mathrm{r}_{2}, \mathrm{r}_{\mathrm{e}}$, minimum, maximum, and effective pore radii, respectively; $\theta_{0}$, half-angle of micropore taper; $\rho, \theta$, running polar coordinates; $z_{i}$, coordinates of the micropore cross section; $S_{i}$, crosssectional area; $\overline{\mathrm{P}}_{\mathrm{i}}$, pressure averaged over the micropore cross section; P , pressure difference on the membrane; $q$, power of the source; $\mu$, dynamic viscosity; $N$, integral number of pores in the membrane; $Q$, volume of liquid flowing through the filter in unit time; $R$, selectivity; $\mathrm{C}_{0}, \mathrm{C}$, concentrations of the filtered substance in the initial preparation and in the filtrate, respectively; D, micropore diameter.

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## INFLUENCE OF THE SHAPE OF A THIN INCLUSION ON THE TEMPERATURE

distribution in a piecewise-homogeneous plane
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The plane stationary problem of heat conduction is considered without taking account of heat elimination through the side surfaces of a composite body with thinwalled interlayers on the interface of materials.

The structure of real materials is always without homogeneity and saturated by defects of the vacancy and impurity type, which often have the form of linear cracks or interlayers. Such inhomogeneities occur not only in the production stage of materials, but are also structural elements in the form of weld or glue connections. Hence, the development of simple and, if possible, exact methods of taking account of the influence of defects on the distribution of physicomechanical fields, particularly the temperature field, is of great value.

A system of $N$ symmetric inclusions of the small thickness 2 h are arranged on the abscissa axis $L=L^{\prime}+L^{\prime \prime}$ of a Cartesian x0y coordinate system so that $L^{\prime}=L_{1}+\ldots+L_{N}$, where $L_{n}=\left[\alpha_{n}, b_{n}\right]$ is the middle line of the $n$-th inclusion. The quantity $h=h(x)$ and $h\left(a_{n}\right)=$ $h\left(b_{n}\right)=0$ on the inclusion end faces. An ideal thermal contact with two half-planes $S_{2}$ and $\mathrm{S}_{1}$ of different thermophysical properties, which are directly in contact on L ", is accomplished along the upper $\mathrm{L}_{2}^{\prime}$ and lower $\mathrm{L}_{1}^{\prime}$ boundaries of the interlayer. The thermal flux $\mathrm{q}_{1}+$ $i q_{2}$ at infinity in the upper half-plane $S_{2}$, the thermal sources of intensity $q_{k}^{\circ}$ at the points $z_{k}=x_{k}+i y_{k}$ of the domains $S_{k}$, and the intensity $q_{0}^{\circ}$ at the point $z_{0}=x_{0}$ on the axis of a

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